Modeling Distribution Options and Constraints for Smart Power Systems with Variable Renewable Energy[1]

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Abstract

We propose a linear programming framework to model distribution network characteristics, and market clearing processes for flexible load and distributed energy resources providing reserve and reactive power compensation. We first show that the Nash equilibrium solution representing the interaction between utility and customers for demand response and distributed reserve transactions can be approximated by a linear program when the players (i.e. the customers) are numerous and tend to become infinitesimal. Then we provide a linear program to reveal the market prices, corresponding to the marginal cost for the utility. The goal in developing this model is to provide a new module for a regional long term model of development of smart energy systems. This module will then introduce in the modelling of energy transition, the new options and constraints that are provided by a penetration of renewables with the possibility of implementing distributed markets for demand response and system services permitted by the development of the cyber-physical layer. A case study of a potential smart urban distribution network in Europe is carried out and provides numerical results that illustrate the proposed framework.

Keywords: Long term energy planning, distribution network, distributed energy sources, intermittency, reactive compensation, secondary reserve.

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1. Introduction

In this paper we propose a linear programming framework to model distributed generation, flexible loads and distributed energy resources (DERs) along with the distribution grid topology and power flow in the context of smart energy systems in the presence of variable renewable energy. This modeling framework enables the "commoditization" of demand response, the introduction of decentralized markets for the optimal scheduling of secondary reserve, storage-like flexible loads and reactive power compensation through the dual use of volt/var control devices that accompany DERs. This linear program has the potential to capture optimal adaptive operating costs, and, as such, provide the operating cost module of a long-term optimal energy technology mix capacity expansion model such as ETEM-SG [3], that is capable of emulating the development of an efficient regional energy system with a planning horizon of 30 to 50 years.

The drive toward sustainable development will be facilitated by the transition to smart energy systems relying on the interface and co-optimization of the cyber and physical layers modeling the Electricity Cyber Physical System (CPS). Increasing penetration of intermittent and volatile renewable energy sources connected at the transmission (e.g. wind farms) or the distribution networks (e.g. roof top PV panels) will impose new operational requirements. Fortunately, the advent of grid friendly Flexible loads and DERs including variable speed drive powered CHP micro-generators [31], heat pumps [35, 26], and electric vehicles [9, 14], provide new opportunities to optimize power systems by providing fast reserves and putting accompanying volt/var control devices (PV inverters, EV chargers and the like) to dual use for reactive power compensation. Under these circumstances, flexible loads and DERs can significantly improve operational and investment efficiencies.

As indicated above, our aim is to incorporate a representation of the constraints along with the significant degrees of freedom and capabilities of these new technology developments at distribution level in a regional long-term multi-service and multi-energy model. Linear programming models for regional energy systems analysis were introduced early on (see e.g. [47, 48, 49]) in conjunction with the development of the MARKAL/TIMES [15] family of models under the aegis of ETSAP, a committee of the IEA. The interest for a regional or local energy modeling capability has been recently strengthened by the development of the smart grid and smart city concepts (see e.g. [5, 23, 22, 24, 27]). The representation of demand-response in the open-source energy model ETEM-SG [12] has been described in [3], and a similar development in TIMES has been proposed in [6], whereas a representation of smart grids has been introduced in the open-source energy modeling kit OSeMOSYS [17, 46].

This paper focuses on incorporating into a linear program, which would be compatible with the aforementioned regional long-term models, the ability of DERs (broadly construed distributed flexible loads, generation and other resources) to provide reserves, reactive power compensation and shift their operation over time so as to reduce losses, congestion, wholesale energy costs and distribution asset (particularly transformer) wear and tear.

Distribution Location Marginal Cost Based Pricing (DLMPs) are attracting increasing attention in the literature. Reactive power costing/pricing was addressed early on in [4] using Alternating Current (AC) load flow models. Several works [16, 42, 44, 30, 33, 43] have addressed DLMPs at various levels of detail but have all modeled congestion using the transmission network paradigm, where transmission line power flow is constrained by each line's capacity. Whereas this may be appropriate for high voltage transmission line networks, in distribution networks, congestion appears in the form of hard voltage magnitude constraints that must be observed at each node/buss, and also as soft constraints at transformers whose life declined rapidly when power flowing through them exceeds their rated capacity and at lines whose marginal losses increase rapidly with power flowing over them. We believe that use of models that capture salient real and reactive power issues and marginal distribution asset life degradation is crucially necessary and appropriate in modeling distribution networks for today's consumption technologies and DERs. As long as real and reactive power and losses are modeled, whether these models are non-linear as is the full AC load flow model or they derive from an appropriately linearized version of the full AC model with repeated recalculation of linearization gaps is an approximation discussion of lesser significance. Indeed, full AC load flows have been used amongst others in a centralized market clearing formulation in [36, 8, 38, 13, 29, 11, 45, 42], although detailed distribution network costs and congestion are addressed only in [36, 38]. Robust-convergence distributed dual decomposition models have been developed recently using Proximal Message Passing (PMP) algorithms. PMP algorithm based models have been applied to very large networks (see [19, 20] and references therein) although most have not modeled reactive power and complex DERs with inter temporally coupled preferences. PMP based distributed DLMP algorithms on a fully developed distribution network model have been reported in [37] and [39].

In this paper, we have developed a tractable linear DLMP model derived from the full AC distribution network model reported in [36, 38]. We use an iterative linearization that recalculates the point about which we linearize till the linearization gap and slope converge. We rarely need more than few iterations to achieve convergence for a reasonably tight convergence tolerance. Our model is capable of representing a daily cycle of a distribution network and of deriving DLMPs and demand response schedules for each period in each of several typical days that we use to derive weighted averages and estimate annual operating costs that are compatible with fully adaptive load and DER behavior. The ultimate goal is to introduce distribution network costs, benefits and adaptability into a regional integrated energy systems analysis model that optimizes the transition costs to environmental sustainability of a grid that relies on variable energy generation, smart grid management and broadly construed demand response. This will be the object of a separate paper.

The proposed approach is consistent with the one advocated by Mathiesen et al. [28], where they show that "... the transition from fossil fuels towards the integration of more and more renewable energy requires rethinking and redesigning the energy system both on the generation and consumption side. Smart Energy System must have a number of appropriate infrastructures for the different sectors of the energy system, including smart electricity grids, smart thermal grids (district heating and cooling), smart gas grids and other fuel infrastructures...". They build prospective scenarios for Denmark 2050, using the CEESA project [25, 32] scenario building tool.

This paper's contribution is the computationally efficient modeling of demand response and the associated reserves [7], as well as the reactive power compensation [18] that can be provided by Distributed Energy Resources and flexible loads. Thus, sufficient introduction of DERs can act synergistically to mitigate the volatility of renewable generation [41] and enable its strong integration into the electricity grid. Moreover it is consistent with the game theoretic approach proposed in [50] as it shows that the LP framework can be justified as a limit of Nash equilibrium solutions, when the players are numerous and tend to have infinitesimal influence on the price.

The paper is organized as follows: In section 2 we show, that under appropriate conditions of grid cost convexity and small, price taker participants, a Linear Program leads to the usual competitive market equilibrium. In section 3 we develop a linear program that represents accurately the market based mechanisms driving optimal demand response in smart grids. A similar approach is used for the representation of distribution network markets for reserves and reactive power, with all three products, real power, reactive power and reserves clearing simultaneously; in section 4 a numerical illustration is given, based on a scenario of strong variable energy penetration in a european region. In section 5 we conclude and discuss further developments envisioned in integrating this distribution grid sensitive model into regional integrated energy systems, like ETEM-SG or OSeMOSYS.

2. Modeling demand response in a linear program

We show that the nonzero sum game describing the relation between a cost minimizing retailer and the set of customers who individually optimize their benefits from real power, reactive power, and reserve transactions leads under appropriate grid cost convexity and participant price taker conditions – i.e. customer demand being arbitrarily small – to an equilibrium which is described by the solution of the linear program. In [2] the relationship between a retailer practising real-time pricing and a finite set of customers optimizing the timing of their electricity consumption is modeled as a non-cooperative game which admits, under some general conditions a unique and stable Nash equilibrium. The model is summarized below, in a slightly more general formulation than the one used in [2]. The Grid Operator (GO) has access to its own production equipment and also to the wholesale market. Depending on the total demand $D(t)$ and the level of reserve $\Re(t)$ that the GO must secure during a time slot t of the day, the marginal cost of production is given by $\gamma(\Re(t), D(t))$, which is the price that will be charged. Each consumer i has a minimum daily requirement of electricity β^i_j for each type of service j. Let $x^i_j(t)$ denote the demand by consumer i for service j at time slot t. The following constraints must thus be satisfied:

$$\sum_t x^i_j(t) \geq \beta^i_j, \tag{1}$$

together with:

$$\begin{aligned} x^i_j(t) &\geq x^i_j[\min](t) && (2)\\ x^i_j(t) &\leq x^i_j[\max](t) && (3) \end{aligned}$$

where $x^i_j[\min](t)$ and $x^i_j[\max](t)$ are given bounds.

The consumer can also use its flexible load to provide reserve to the GO. Let $r_j^i(t)$ denote the contribution to reserve by consumer i using the flexible load technology (e.g. PHEV or heat pump) providing service j at time slot t. A capacity constraint must hold for each i, j, t:

$$r_j^i(t) \leq \delta_j^i(\zeta_j^i(t) - \alpha_j x_j^i(t)), \tag{4}$$

$$\tag{5}$$

where $\zeta_j^i(t)$ is the available capacity at time slot t.

Let us define the following vectors: $\underline{\mathbf{x}} = (\mathbf{x}(t))_{t\in T}$, where $\mathbf{x}(t) = (\mathbf{x}^i(t))_{i\in I}$, with $\mathbf{x}^i(t) = (x_j^i(t))_{j\in J}$, and, similarly, $\underline{\mathbf{r}} = (\mathbf{r}(t))_{t\in T}$, where $\mathbf{r}(t) = (\mathbf{r}^i(t))_{i\in I}$, with $\mathbf{r}^i(t) = (r_j^i(t))_{j\in J}$.

We assume that this total contribution will never exceed $\Re(t)$. This is the case, if $\sum_{j,i} \delta_j^i \zeta_j^i(t)$ is much smaller than $\Re(t)$. The total demand in time slot t is given by:

$$D(t) = \sum_i \sum_j x_j^i(t), \tag{6}$$

and the total contribution to reserve is given by

$$R(t) = \sum_i \sum_j r_j^i(t). \tag{7}$$

This determines the marginal cost $\gamma(\Re(t) - R(t), D(t))$, and hence the tariff payed at time slot t by each customer. The aim of the i-th customer is thus to minimize

$$\psi^i(\underline{\mathbf{x}}, \underline{\mathbf{r}}) = \sum_t \gamma(\Re(t) - R(t), D(t)) \left(\sum_j x_j^i(t) \right), \tag{8}$$

given the actions taken by the other agents and under the constraints (1) to (4). Note that the interdependence among customers comes from the price determination equation, see (6). The dual variables corresponding to the constraints (1) to (4) respectively are denoted η_j^i, $\mu_j^i(t)$, $\nu_j^i(t)$ and $\pi_j^i(t)$; they are greater or equal to zero. The Lagrangian for the i-th consumer can thus be written as:

$$
\begin{aligned}
\mathcal{L}^i \quad = \quad & \psi^i(\underline{\mathbf{x}}, \underline{\mathbf{r}}) + \sum_j \eta_j^i \left(\beta_j^i - \sum_t x_j^i(t) \right) \\
& + \sum_{t,j} \mu_j^i(t) \Big(x_j^i[\min](t) - x_j^i(t) \Big) \\
& + \sum_{t,j} \nu_j^i(t) \Big(x_j^i(t) - x_j^i[\max](t) \Big) \\
& + \sum_{t,j} \pi_j^i(t) \Big(r_j^i(t) - \delta_j^i(\zeta_j^i(t) - \alpha_j x_j^i(t)) \Big).
\end{aligned} \tag{9}
$$

The first order conditions for a Nash equilibrium are given by:

$$
0 \quad = \quad \frac{\partial \mathcal{L}^i}{\partial x_j^i(t)} = \frac{\partial \psi^i(\underline{\mathbf{x}})}{\partial x_j^i(t)} - \eta_j^i - \mu_j^i(t) + \nu_j^i(t) - \pi_j^i(t)\, \delta_j^i\, \alpha_j, \tag{10}
$$

$$
0 \quad = \quad \frac{\partial \mathcal{L}^i}{\partial r_j^i(t)} = \frac{\partial \psi^i(\underline{\mathbf{r}})}{\partial r_j^i(t)} + \pi_j^i(t). \tag{11}
$$

which can be written as:

$$
\begin{aligned}
0 \quad = \quad & \gamma\big(\Re(t) - R(t), D(t)\big) + \gamma'_D\big(\Re(t) - R(t), D(t)\big) \sum_k x_k^i(t) \\
& -\eta_j^i - \mu_j^i(t) + \nu_j^i(t) - \pi_j^i(t)\, \delta_j^i\, \alpha_j
\end{aligned} \tag{12}
$$

$$
0 \quad = \quad \gamma'_{\Re}(\Re(t) - R(t), D(t)) - \pi_j^i(t). \tag{13}
$$

with the following complementarity conditions:

$$
\eta_j^i \geq 0 \qquad \text{and} \qquad \eta_j^i \Big(\sum_t x_j^i(t) - \beta_j^i \Big) = 0, \tag{14}
$$

$$
\mu_j^i(t) \geq 0 \qquad \text{and} \qquad \mu_j^i(t) \Big(x_j^i(t) - x_j^i[\min](t) \Big) = 0, \tag{15}
$$

$$
\nu_j^i(t) \geq 0 \qquad \text{and} \qquad \nu_j^i(t) \Big(x_j^i(t) - x_j^i[\max](t) \Big) = 0 \tag{16}
$$

$$
\pi_j^i(t) \geq 0 \qquad \text{and} \qquad \pi_j^i(t) \Big(r_j^i(t) - \delta_j^i(\zeta_j^i(t) - \alpha_j x_j^i(t)) \Big) = 0. \tag{17}
$$

Applying classical theorems (see *e.g.* [40]), we can easily find conditions which assure that an equilibrium exists and that it is unique if the $\gamma(\Re(t), D(t))$

function is strictly convex and increasing in both argument. Notice also that conditions (17 and (13) lead to the conclusion that each player contributes all the available capacity remaining in each of the flexible load to the reserve requirement.

We model now a situation where there are many small agents. To do so, let us assume that each customer i is replicated n times with demand parameters β_j^i/n, capacity parameter $\zeta_j^i(t)/n$ and bounds $x_j^i[\min](t)/n$ and $x_j^i[\max](t)/n$. This describes a game where the number of players increases while the influence of each player diminishes. The first order conditions for a Nash equilibrium (12) are now given by:

$$\begin{aligned} 0 &= \gamma\big(\Re(t) - R(t), D(t)\big) + \gamma'_D\big(\Re(t) - R(t), D(t)\big)\frac{\sum_k x_k^i(t)}{n} \\ &\quad -\eta_j^i - \mu_j^i(t) + \nu_j^i(t) - \pi_j^i(t)\,\delta_j^i\,\alpha_j \qquad (18) \\ 0 &= \gamma'_{\Re}(\Re(t) - R(t), D(t)) - \pi_j^i(t). \qquad (19) \end{aligned}$$

When $n \to \infty$ the conditions of a competitive equilibrium are met. Note that for each type of player i and type of service j, the following constraint must hold:

$$\sum_t \frac{x_j^i(t)}{n} - \frac{\beta_j^i}{n} \geq 0, \qquad (20)$$

which is the same as (1). The same reasoning applies for the other constraints. As a consequence, the KKT multipliers are the same as before.

In the large n limit, the term $\gamma'_D\big(\Re(t) - R(t), D(t)\big)\frac{\sum_k x_k^i(t)}{n}$ tends to 0, the condition (13) thus becomes:

$$\gamma\big(\Re(t) - R(t), D(t)\big) - \eta_j^i - \mu_j^i(t) + \nu_j^i(t) - \pi_j^i(t)\,\delta_j^i\,\alpha_j = 0. \qquad (21)$$

Each consumer is now a price taker. His decisions have no influence on the price. The quantities $x_j^i(t)$ are then determined by using (21) together with the complementarity conditions (14)-(16). As before, each agent uses all the unused available capacity of its flexible load to provide reserve.

Remark 1. *Here we make the simplifying assumption of a sequential decision, i.e. the agent first decides on energy usage and then offers excess capacity to reserves. This is compatible with sequential market clearing which was indeed the case in advanced markets in the US. Now, energy and reserves are cleared in "joint" markets, where it is indeed possible, depending on the price of reserves relative to the price of energy for an agent to trade off energy for reserves.*

In order to represent the GO cost, a linear program is particularly suitable. There are m generation facilities (indexed by κ), n demand blocks (indexed by θ). The model is characterized by the following parameters:

- Number of hours in demand block θ : H_θ,
- Cost per produced MWh by facility κ : c_κ,
- Capacity in MW of facility κ : K_κ.

Let $z_{\kappa\theta}$ be the energy flowing from facility κ during the time slot θ. Let $y_{\kappa\theta}$ be the contribution to reserve from facility κ during the time slot θ. If the timing of demands of type j for consumer type i were under direct control of the retailer, it would solve the following linear program:

$$\underset{\{z_{\kappa\theta},\, x_j^i(\theta)\}}{\text{minimise}} \sum_{\theta=1}^{n} \sum_{\kappa=1}^{m} c_\kappa z_{\kappa\theta} \tag{22}$$

under the following constraints:

$$\begin{aligned}
\sum_{\kappa=1}^{m} z_{\kappa\theta} - \sum_{i,j} x_j^i(\theta) &\geq 0, && (23)\\
-z_{\kappa\theta} &\geq -(K_\kappa - y_{\kappa\theta})H_\theta, && (24)\\
K_\kappa - y_{\kappa\theta} &\geq 0 && (25)\\
z_{\kappa\theta} &\geq 0, && (26)\\
\sum_{\kappa=1}^{m} y_{\kappa\theta} + \sum_{i,j} r_j^i(\theta) &\geq \Re(\theta), && (27)\\
y_{\kappa\theta}, r_j^i(\theta) &\geq 0, && (28)\\
\sum_{\theta} x_j^i(\theta) &\geq \beta_j^i, && (29)\\
x_j^i(\theta) &\geq x_j^i[\min](\theta), && (30)\\
x_j^i(\theta) &\leq x_j^i[\max](\theta). && (31)\\
r_j^i(t) &\leq \delta_j^i(\zeta_j^i(t) - \alpha_j x_j^i(t)). && (32)
\end{aligned}$$

These constraints correspond to the demand having to be met (23), the available capacity after contribution to reserve of facility κ (24) and the energy flows having to be positive (26). The dual variables for equations

(23), (24), (27), (29), (30), (31) and (32) are respectively given by ϖ_θ, $\vartheta_{\kappa,\theta}$, ω_θ, η^i_j, $\mu^i_j(\theta)$, $\nu^i_j(\theta)$ and $\pi^i_j(\theta)$. By applying the optimality condition for a linear program and if the variables $x^i_j(\theta)$ are in the optimal basis, meaning that their reduced costs must be zero, we get the following:

$$\nu^i_j(\theta) - \mu^i_j(\theta) - \eta^i_j - \pi^i_j(\theta)\,\delta^i_j\,\alpha_j + \varpi_\theta = 0. \tag{33}$$

This equation is to be compared with (21). The dual variable ϖ_θ corresponds to the marginal production cost to satisfy demand. Therefore we conclude that (33) and (21) are identical. If the $r^i_j(\theta)$ are in the basis, the following reduced cost must be equal to zero

$$\pi^i_j(\theta) + \omega_\theta = 0. \tag{34}$$

Because reserve has to be provided, ω_θ is strictly negative, so $\pi^i_j(\theta)$ must be strictly positive and the corresponding constraint is active, for all i, j, θ, which means that each category of consumer will use all its spare capacity to provide reserve.

Henceforth, *the solution of the linear program gives also the optimal response of consumers to a marginal cost reflecting prices.* We could develop similar arguments for the representation of optimal response of users to market based incentives to provide decentralized reactive power compensation. This gives a theoretical justification for the linear programming approach that we propose in the next sections.

Remark 2. *We wish to conclude by noting that the proof given above is based on a power system where the actual Transmission and Distribution Network is not modeled, and hence line losses and load flow constraints are assumed to be negligible. Although these assumptions are reasonable and allow us to prove the equilibrium existence in a succinct and elegant manner, it should be noted that they are not necessary. In fact, a similar result, namely the existence of a unique and stable Nash equilibrium, has been shown for price taking agents that see energy and reserve prices which are sensitive to their exact location in their network. The marginal cost – and hence price – at a particular location during the same time slot t may differ across locations because of the non-linear AC load flow relationships and location specific marginal line losses. For example, an agent located at node n and consuming energy $d_n(t)$ while providing reserves $r_n(t)$, in fact consumes energy $(d_n(t)[1 + \texttt{marginallosses}_n(t)])$ and provides reserves $(r_n(t)[1 + \texttt{marginallosses}_n(t)])$, and hence may prefer to see higher* *marginal losses if the price of reserves is higher than the price of energy. In [7] it is shown that whereas a stable Nash Equilibrium exists, it may differ from the socially optimal equilibrium, converging to it asymptotically as the size of reserves offered at location n is negligible relative to the inflexible demand at that node, where inflexible demand is demand that can not trade off energy for reserves.*

3. A linear program to represent distribution network markets for demand response, reserves and reactive power

3.1. Toward a representation of demand response and other distribution network markets in global energy models

Global energy models, like MARKAL [15], MESSAGE [34] or TIMES [21], address, for a given region, an optimization of a reference energy system, including extraction and source of primary energy, import and export of various energy forms, conversion and process for the production of final energy and choice of demand devices to provide the useful energy (demand for energy services). The planning horizon is generally long enough to offer a possibility for the energy system to have a complete investment technology mix turnover (45 years for MARKAL, and even 100 years for the use of TIMES and MESSAGE in the assessment of climate policies). In these models the needed adjustment of production to demand for non-storable energy forms, like electricity (ELC) or low temperature heat (LTH), is represented through the introduction of time slices that divide each year and permit an approximation of the load curve. For a given time slice, the fraction of the demand which falls in this time slice is a key parameter. Recently, several models have been proposed to include demand response, and more generally, smart grids, in global energy models [3, 5, 46]. In these approaches the demand response and grid storage activities are modeled within the time structure defined by the time slices of the global energy model. This necessitates, nevertheless the introduction of a new set of constraints to model the operations at the distribution level. Note that the introduction of new elements in the model is facilitated in open-source codes such as OSeMOSYS [17] or ETEM [12]. In the rest of this section we propose a complete sub-model that describes the costs and benefits optimized subject to distribution network load flow and other constraints. Intermittent production from renewables, grid storage and demand response in a regional energy system are modeled and represented by constraints. In section 4, the aforementioned sub-model is tested on realistic input data consistent with a case study inspired by a recent energy plans of a Swiss region [1].

3.2. *A linear program to represent the optimal operation of a smart distribution grid*

In this section, we follow the general principles of section 2 and propose a linear programming representation of the optimal scheduling of centralized and distributed loads, storage and generation units for a local/regional power system. Figure 1, below, summarizes the simplified topology of the distribution system that we consider in our case study. The ∞-bus (b_∞) corresponds

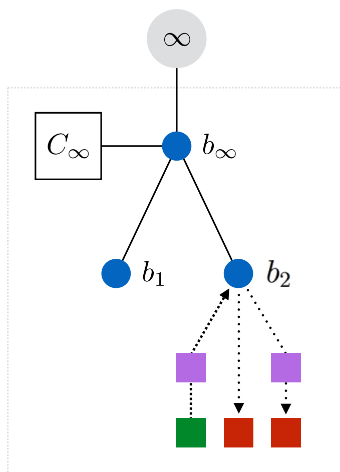

Figure 1: Representation of the power network. Circles denote buses, squares represent power electronics, flexible loads and distributed generators.

to the substation and each downstream bus (b_1 and b_2) corresponds to loads and DERs connected to a distribution feeder. The model's logic is as follows: At bus ∞ there are conventional generators and wind generators. The production of wind generators is exogenously defined and incurs no variable cost. There are n distribution feeders connected to bus ∞. Each feeder bus hosts (i) demand corresponding to conventional loads (typically lighting), which consumes as a by-product "reactive power" whose magnitude depends on a constant power factor, (ii) flexible loads (typically EV battery charging, variable speed drive heat pumps for space conditioning), and (iii) PV generation. EV battery chargers and PV inverters can provide reactive power compensation as needed when they have excess capacity, i.e. when the sun does not shine or when the EV battery is not charging. During a given time slice, flexible loads produce value (or utility to their owners) by providing a service, such as space conditioning that maintains inside temperature within a comfort temperature zone, increasing the state of Charge of thee EV battery and the like. Although in principle other types of reserves can be also modeled, we focus on secondary reserves made necessary by renewable generation and uncertainty in conventional loads and generation. The reserve required by the system operator can be provided by conventional centralized generators but also by the flexible loads, in particular by the PHEV/EVs. When the apparent power flowing through a feeder's transformer rises close to or exceeds its rated capacity, the transformer's life degrades rapidly contributing to distribution network's variable costs. High apparent power flow is also associated with high distribution line flows. Reactive power compensation decreases the apparent power flow providing significant cost reduction through lower energy losses and transformer life degradation. In addition, requiring less reactive power at bus ∞, reduces further the grid opportunity cost associated with the provision of reactive power compensation at the substation. The production of energy by conventional generators generates a cost (cost of fuel and variable operation cost per kWh. The model optimizes real power, reactive power and reserves associated with each participant so as to minimize grid costs and participant costs minus benefits, subject to load flow, voltage, energy balance and reserve requirement constraints.

3.2.1. Main assumptions

Assumption 1. The transmission network is made up of a single bus, i.e. transmission lines connecting centralized generators G_k, $k \in \{1, \dots, K\}$, to the bus that supplies all distribution substations have negligible resistance.

Assumption 2. There are N distribution feeders denoted by n with $n \in \{1, \dots, N\}$. Each distribution feeder is represented by a single aggregated line and a single transformer with all of the demand and distributed resources and generation at the end of the line. The aggregated line has parameters R_n and X_n representing the line's aggregate resistance and reactance ($X_n = 2\pi f L_n$ where f is the frequency – 50Hz –, and L_n is the line's in series inductance).

The line transformer has rated capacity for apparent power flow[2] S_n.

Assumption 3. The year is represented by a small number of typical days, say 6 days corresponding to the three seasons (Winter, Summer, Spring-Fall) and two week day types (working weekday, weekend-Holiday). Each day is subdivided into T groups of hours, called timeslices or time slots denoted by t with $t \in \{1, \dots, T\}$. δ_t is the duration of timeslice t expressed as a fraction of the year.

Assumption 4. Demand for energy services and ability for distributed generation or resource provision is specified as follows:

(i) for conventional demand at feeder n, such as lights or non-storage/thermal demand, real power demand is denoted $D_n(t)$, and reactive power demand[3] is $Q_n(t) = \gamma_n D_n(t)$; it is specified for each group of hours t.

(ii) For *flexible/storage like loads*, such as thermal storage buildings, space heating/conditioning, electric vehicles and the like, it is specified for the whole day or for an aggregation of several periods.

(iii) For distributed generation (such as PV) the output is known[4] for each period t, i.e. $g_n^{PV}(t)$, is an exogenous input.

(iv) For distributed resources that accompany electric vehicles or PV generation, inverters and converters that are embodied can produce reactive power using excess capacity that they may have. For example, a PV with capacity $\bar{g}^{PV}$ producing during time t at 60% of its installed capacity can produce

$$Q^{PV}(t) = \sqrt{(\bar{g}^{PV})^2 - (0.6\bar{g}^{PV})^2},$$

if needed to compensate for the reactive power consumed by inductive or other loads with a less than 1 power factor. Note that since the

[2]The apparent power is the norm of the complex power vector (real part is active power, imaginary part is reactive power.)

[3]The parameter $\gamma_n = \tan(\phi_n)$ is a fixed factor taking value in the range of .3 to .577 to .70 as ϕ ranges from 17 to 30 to 35 degrees. Today, residential loads have $\phi \sim 30$ degrees. Note that normal power factor ballast neon lights have a ϕ of 50 to 60 degrees!

[4]In an advanced formulation the output will be a random function of t with known probability measure.

production of real power of the PV is known, i.e. set exogenously during a period t, the maximum reactive power it can produce is also known and can be expressed as a linear inequality. The approximation introduced, is simply that there is no capability to decide to produce less from the PV installation than the solar irradiation allows during period t in order to provide more reactive power compensation.

Assumption 5. For flexible loads that also have power electronics/inverters, reactive power compensation capabilities will be limited to a linear constraint,

$$Q^F(t) \leq \bar{D}^F - D^F(t),$$

which will be exact when $D^F(t) = 0$. This is not too bad since flexible loads, with a notable exception when they offer *down secondary reserves,* will be consuming either at maximum power $\bar{D}^F$ or at 0.

Assumption 6. Loss of life (number of hours of economic life lost per hour of clock time) of the aggregated transformer in radial feeder n depends on the ambient temperature and the apparent power flowing through it. It is equal to

$$\Gamma_n(\theta_n^{Amb}(t), S_n^{\infty}(t)) = e^{\frac{15000}{383} - \frac{15000}{273+\theta_n^H(t)}}$$

where

$$\theta_n^H(t) = \theta_n^{Amb}(t) + k_{1,n} + k_{2,n} \left(\frac{S_n^{\infty}(t)}{S_n^N} \right)^2,$$

with

$$(S_n^{\infty}(t))^2 = (D_n^{\infty}(t))^2 + (Q_n^{\infty}(t))^2,$$

and S_n^N the nominal or rated capacity of the aggregate transformer in radial line n; $\theta_n^H(t)$ is the the hottest spot temperature with the values of the constant coefficients $k_{1,n}$ and $k_{2,n}$ estimated from the relationships:

$$110 = 25 + k_{1,n} + k_{2,n} \left(\frac{S_n^N}{S_n^N} \right)^2$$

and

$$180 = 25 + k_{1,n} + k_{2,n} \left(\frac{1.55\, S_n^N}{S_n^N} \right)^2.$$

Assumption 7. Generator ramp constraints are negligible.

3.2.2. Quantities involved and their definition

v_n**:** Tension in feeder n. The units are chosen so that the tension is normalized with value equal to 1.

$D_{n,i}(t), D^F_{n,i}(t), Q_{n,i}(t), Q^F_{n,i}(t)$**:** Real power of conventional and flexible load, reactive power conventional and flexible loads. There may be multiple loads at each location n, $i = 1, 2, \ldots, i(n)$ each with its own utility. All are decision variables except for $Q_{n,i}(t)$ which is a fixed multiple of $D_{n,i}(t)$,

$$Q_{n,i}(t) = \gamma_{n,i}(t) D_{n,i}(t), \quad \text{for} \quad \gamma_{n,i}(t) \quad \text{given},$$
$$-[\bar{D}^F_{n,i}(t) - D^F_{n,i}(t)] \leq Q^F_{n,i}(t) \leq \bar{D}^F_{n,i}(t) - D^F_{n,i}(t).$$

Also, in general $F \in \{\sigma, V\}$ two types of flexible loads, space heating/conditioning through varying speed heat-pumps and electric vehicles[5].

$\bar{D}_{n,i}(t), \bar{D}^F_{n,i}(t)$**:** Maximal consumption of conventional and flexible load[6].

$Q^\infty_n(t)$ is the uncompensated reactive power consumed in the distribution feeder n plus uncompensated reactive power losses over the distribution lines which has to be compensated at the substation.

$D_n(t), D^F_n(t), Q_n(t), Q^F_n(t)$**:** where $D_n(t) = \sum_{i=1}^{i(n)} D_{n,i}(t)$ and similarly for $D^F_n(t)$, $Q_n(t)$, $Q^F_n(t)$.

$\Re^V_n(t), \Re^\sigma_n(t)$:Reserves provided during period t by electric vehicles at n and by space heating/conditioning at n.

$G_k(t), \Re_k(t), \bar{G}_k, \underline{G}_k$: Power and reserves provided by centralized generator k during hour group t, its capacity, and its minimum generation level. The latter two are input parameters.

c_k, r_k : variable cost per PJ and cost per PJ of reserves offered by centralized generator k.

[5]The list can be of course expanded.

[6]Note that $D_{n,i}(t) = \bar{D}_{n,i}(t)$ since conventional loads are inelastic and hence can be considered fixed.

$\pi^{\Re,\infty,0}(t)$: Opportunity cost for reactive power generation from centralized generators. It corresponds to the selling price of reserve, or the trading price of electricity on the transmission grid.

$W_k(t), \bar{W}_k$: Centralized wind generation k expressed as a fraction of its installed capacity $\bar{W}_k$ during hour group t, an exogenously set variable representing prevalent wind during t at k.

$g_n^{PV}(t), \bar{g}_n^{PV}$: Distributed generation of PV installed in feeder n, and its capacity. Both are exogenously specified.

$Q_n^{PV}(t)$: Reactive power output of PV facility at n during hour group t.

$\Re(t) + \sum_k \xi_k W_k(t)$: System reserves needed during hour group t to cover load and wind generation variability. The ξ_k's are given.

$x_n(t)$: State of charge of EVs at radial line n at period t.

$\Delta x_n(t)$: The demand for transport to be delivered by EV batteries at period t.

$\underline{\theta}_n(t) \leq \theta_n(t) \leq \bar{\theta}_n(t)$: inside temperature of space conditioned facilities at radial line n during hour group t and its comfort zone.

$\eta_n^{gain}(t), \eta_n^{loss}(t)$: coefficients of heat gain or loss at radial line n during period t.

$\theta_n(t)^{Ambient}$: Ambient temperature in radial distribution n during period t.

3.2.3. Performance criterion

We define here the subproblem of minimizing the power system operation cost for a given typical day. In ETEM-SG this will be integrated in the whole expression of the system cost over all periods, typical days and time-slices.

$$\min_{G,\Re,D,D^\theta,D^V,\Re^\theta,\Re^V,Q^\theta,Q^V} \sum_t \delta_t \eta \Bigg\{ \pi^{\Re,\infty,0}(t) \left[\bar{G}^\infty - \sqrt{(\bar{G}^\infty)^2 - (Q^\infty(t))^2} \right] + \sum_{k=1}^{K} (c_k G_k(t) + r_k \Re_k(t)) + \sum_n c_n^{tr}\ \Gamma_n(\theta_n^{Amb}(t), S_n^\infty(t)) \Bigg\} \quad (35)$$

where $\eta = 31.536$ is the conversion factor from GW to PJ per Year and δ_t is the timeslice t duration, expressed in fraction of year. Let us detail each term of the model

First term. $\delta_t\eta\pi^{\Re,\infty,0}(t)\left[\bar{G}^\infty - \sqrt{(\bar{G}^\infty)^2 - (Q^\infty(t))^2}\right]$. There is an "opportunity cost" incurred by the distribution substation generator due to the provision of reactive power $Q^\infty(t) = \sum_n Q_n^\infty(t)$. A generator with capacity C^∞ can produce real and reactive power, $P^\infty(t)$ and $Q^\infty(t)$, while respecting the constraint

$$(P^\infty(t))^2 + (Q^\infty(t))^2 \leq (C^\infty)^2. \tag{36}$$

Real power cannot be negative, while reactive can be either positive or negative. Positive $Q^\infty(t)$ indicates generation and negative $Q^\infty(t)$ indicates consumption of reactive power. When the high voltage clearing price of real power $\pi^{\Re,\infty,0}(t)$ is larger than the fuel cost of the generator, the generator wishes to generate real power at its full capacity and sell it to the high voltage market. However, due to the fact that it must generate reactive power, and the constraint (36) above, it is forced to generate less real power, namely

$$P^\infty(t) \leq \sqrt{(C^\infty)^2 - (Q^\infty(t))^2}. \tag{37}$$

This results in decreasing its potential generation of real power by

$$C^\infty - \sqrt{(C^\infty)^2 - (Q^\infty(t))^2}$$

and losing income

$$\delta_t\eta(\pi^{\Re,\infty,0}(t) - c^\infty)\left(C^\infty - \sqrt{(C^\infty)^2 - (Q^\infty(t))^2}\right)$$

adjusted by the fact that the generator will also save fuel cost by virtue of the fact that it will generate less real power. We assume here that the fuel cost c^∞ is negligible.

Second term. $\sum_{k=1}^K \delta_t\eta(c_k G_k(t) + r_k\Re_k(t))$. It is the sum for all centralized generator k of providing energy output $\delta_t\eta G_k(t)$ and reserve $\delta_t\eta\Re_k(t)$.

Third term. $\sum_n c_n^{tr}\delta_t\eta\,\Gamma_n(\theta_n^{Amb}(t), S_n^\infty(t))$. It represents the cost associated with the loss of life duration for the transformers at each feeder, which is a function of the apparent power flow $S_n^\infty(t) = \sqrt{(P_n^\infty(t))^2 + (Q_n^\infty(t))^2}$ through them and the ambient temperature.

Linearized version. The linearized version is obtained through a Taylor development in the neighborhood of the optimal solution. Therefore, an update procedure should be implemented when using the linearized version.

$$\min_{G,\Re,D,D^\theta,D^V,\Re^\theta,\Re^V,Q^\theta,Q^V} \sum_t \delta_t \eta \Bigg\{ \pi^{\Re,\infty,0}(t)[\bar{G}^\infty - \sqrt{(\bar{G}^\infty)^2 - (Q^{\infty,0}(t))^2} +$$
$$\frac{Q^{\infty,0}(t)(Q^\infty(t) - Q^{\infty,0}(t))}{\sqrt{(\bar{G}^\infty)^2 - (Q^{\infty,0}(t))^2}}] + \sum_k (c_k G_k(t) + r_k \Re_k(t)) + \sum_n c_n^{tr} \left\{ \Gamma_n(\theta_n^{Amb}(t), S_n^{\infty,0}(t)) + \right.$$
$$\left. \left. \sum_n \frac{\partial \Gamma_n(\theta_n^{Amb}(t), S_n^\infty(t))}{\partial S_n(t)} |_{S_n^{\infty,0}(t)} \frac{P_n^{\infty,0}(t)(P_n^\infty(t) - P_n^{\infty,0}(t)) + Q_n^{\infty,0}(t)(Q_n^\infty(t) - Q_n^{\infty,0}(t))}{\sqrt{(P_n^{\infty,0}(t))^2 + (Q_n^{\infty,0}(t))^2}} \right\} \right\}$$

3.2.4. Constraints

Definition of aggregate loads.

Real power load at feeder n

$$P_n(t) = \sum_i D_{n,i}(t) + D_n^V(t) + D_n^\theta(t) - g_n^{PV}(t), \quad \forall t \tag{38}$$

Reactive power to be compensated at feeder n bus

$$Q_n(t) = \sum_i \gamma_{n,i}(t) D_{n,i}(t) - Q_n^V(t) - Q_n^\theta(t) - Q_n^{PV}(t), \quad \forall t \tag{39}$$

$$Q_n(t) \geq 0 \quad \forall t \tag{40}$$

Reserve provided by flexible load at feeder n

$$\Re_n(t) = \Re_n^\theta(t) + \Re_n^V(t) \quad \forall t \tag{41}$$

Real power load at bus ∞ due to feeder n

$$P_n^\infty(t) = P_n(t) + \frac{R_n}{(v_n)^2}\left\{(P_n(t))^2 + (Q_n(t))^2\right\} \quad \forall t \tag{42}$$

Reactive power to be compensated at bus ∞ due to feeder n

$$Q_n^\infty(t) = Q_n(t) + \frac{X_n}{(v_n)^2}\left\{(P_n(t))^2 + (Q_n(t))^2\right\} \quad \forall t \tag{43}$$

Total active power load at bus ∞

$$P^\infty(t) = \sum_n P_n^\infty(t) \geq 0 \quad \forall t \tag{44}$$

Total reactive power to be compensated at bus ∞

$$Q^\infty(t) = \sum_n Q_n^\infty(t) \geq 0 \quad \forall t \tag{45}$$

Linearized versions of Eqs (42)-(43). The linearized version is obtained through a Taylor development in the neighborhood of the optimal solution. Here again an update scheme will be implemented when using the linearized version.

$$
\begin{aligned}
P_n^{\infty}(t) &= P_n(t) + \frac{R_n}{(v_n)^2}\left\{(P_n^0(t))^2 + (Q_n^0(t))^2 + 2P_n^0(t)[P_n(t) - P_n^0(t)] + \right. \\
&\quad \left. 2Q_n^0(t)[Q_n(t) - Q_n^0(t)]\right\} \quad \forall t \qquad (46) \\
Q_n^{\infty}(t) &= Q_n(t) + \frac{X_n}{(v_n)^2}\left\{(P_n^0(t))^2 + (Q_n^0(t))^2 + 2P_n^0(t)[P_n(t) - P_n^0(t)] + \right. \\
&\quad \left. 2Q_n^0(t)[Q_n(t) - Q_n^0(t)]\right\} \quad \forall t \qquad (47)
\end{aligned}
$$

Power balance.

Real power at bus ∞ is provided by wind turbines and other generators

$$P^{\infty}(t) = \sum_k G_k(t) + \sum_\ell W_\ell(t) \quad \forall t. \qquad (48)$$

Reserve requirements.

Reserve provided by centralized and flexible loads meet the demand for reserve

$$\sum_k \Re_k(t) + \sum_n (1 + \tfrac{2R_n P_n^0(t)}{(v_n)^2})\Re_n(t) \geq \Re(t) + \sum_\ell \xi_\ell W_\ell(t) \quad \forall t. \qquad (49)$$

Capacity constraints.

Real power supply at bus ∞ is bounded by capacity limits and reserve requirement

$$\underline{G}_k + \Re_k(t) \leq G_k(t) \leq \bar{G}_k - \Re_k(t) \quad \forall t. \qquad (50)$$

Flexible demand at feeder n is bounded

$$0 \leq D_n(t) \leq \bar{D}_n \quad \forall t. \qquad (51)$$

Bounds on reserve provided by flexible loads

$$\Re_n^V(t) \leq D_n^V(t) \leq \bar{D}_n^V(t) - \Re_n^V(t) \quad \forall t, \qquad (52)$$

$$\Re_n^\theta(t) \leq D_n^\theta(t) \leq \bar{D}_n^\theta(t) - \Re_n^\theta(t) \quad \forall t. \qquad (53)$$

Bounds on reactive power compensation provided by flexible loads

$$0 \leq Q_n^V(t) \leq \bar{D}_n^V(t) - D_n^V(t) \quad \forall t, \qquad (54)$$

$$0 \leq Q_n^\theta(t) \leq \bar{D}_n^\theta(t) - D_n^\theta(t) \quad \forall t. \qquad (55)$$

State equations.

Dynamics and bounds for indoor temperature

$$\theta_n(t+1) = \theta_n(t) + \eta^{loss}_{n,t}(\theta^{Ambient}(t) - \theta_n(t)) - \eta^{gain}_{n,t} D^{\theta}_n(t) \quad \forall n, t, \tag{56}$$

$$\underline{\theta}_n(t) \leq \theta_n(t) \ \leq \ \bar{\theta}_n(t) \quad \forall n, t, \tag{57}$$

$$\theta_n(0) = \theta^0_n. \tag{58}$$

Dynamics for state of discharge of EV's

$$x_n(t+1) = x_n(t) - \delta_t \eta \, D_n(t) + \Delta x_n(t) \quad \forall n, t, \tag{59}$$

$$0 \leq x_n(t) \leq \bar{x}^t_n. \tag{60}$$

4. Illustrative Numerical Results

We illustrate the linear programming based distribution-market clearing formulation developed in section 3 by implementing it on a case study where the regional energy/technology/environment model, ETEM-SG, is calibrated to represent the Léman region in Switzerland. For the sake of demonstration, we consider only two distribution feeders as described in Table 2. Loads are modeled as being connected to these two feeders. This is, of course, an approximation that retains salient qualitative features but does not correspond to actual distribution network topology.

Time structure. We focus on a typical winter daily cycle corresponding to future year 2030, and model it by four unequal length time-slices denoted by WN, WP1, WM, and WP2, respectively. The fraction of the year represented by each time-slice is shown in Table 1 below.

Table 1: Duration of Winter Day timeslices

Timeslices	Fraction of the year	Time interval
WN: Winter night	0.146	12pm - 7am
WP1: Winter morning peak	0.104	7am - 12am
WM: Winter mid-day	0.104	12am - 5pm
WP2: Winter evening peak	0.146	5pm - 12pm

Grid parameters. The following key parameters are used to describe the power system:

- System secondary reserve parameter : $\Re(t) = 0.10$ GW;
- System reserve coefficient to cover wind generation variability (proportion of centralized wind generation) $\xi_k \equiv 0.5$;
- Factor representing reactive power as a proportion of active power consumed by conventional inflexible loads $\gamma_n \equiv 0.3$. This corresponds to a phase angle $\phi = 17^o$ and power factor $\cos(\phi) = 0.956$;
- Variable cost of reserves at the transmission node 0.002 M\$/GW;
- Min inside temperature of space heating/conditioning facilities 18^{o}C;
- Max inside temperature of space heating/conditioning facilities 22^{o}C;

Table 2: Feeder characteristics

Feeder	Description	Resistance	Reactance
F1	Industrial zone	0.040	0.100
F2	Residential zone	0.031	0.045

Table 3: Parameters for space heating (^{o}C)

	Ambient temperature	Heat gain	Heat loss
WN	0	750	0.15
WP1	5	500	0.1
WM	10	500	0.1
WP2	5	750	0.15

Capacities and loads from ETEM-SG. We assume that the scenario run by ETEM-SG has determined year 2030 technology mix and capacities reported in Table 4 and costs as shown in Table 5.

Table 6 shows the demand devices that are generating loads and Table 7 gives the detail of the loads that are connected at each feeder.

Table 4: Installed capacities and availability factors (1 – Forced Outage Rate) for power generation

Centralized Conversion Technologies	(GW)	Availability Factors			
		WN	WP1	WM	WP2
IMP Electricity imports	∞	1	1	1	1
E01 Hydroelectric VAUD	0.184	0.34	0.34	0.34	0.34
E02 Hydroelectric GENEVA	0.170	0.6	0.6	0.6	0.6
E00 Veytaux pumped storage	0.2625	0	1	1	1
E08 Wind	0.1	0.247	0.239	0.242	0.259
Decentralised Generation Technologies	(GW)	Availability Factors			
		WN	WP1	WM	WP2
E07 PHV solar panels	0.013	0	0.075	0.084	0
RCC CHP combined heat power	0.35 (El.)	0.97	0.486	0.323	0.162

Table 5: Variable generation cost M$/PJ

Centralized Conversion Technologies	WN	WP1	WM	WP2
IMP Electricity imports	50	90	90	90
E0F Gas CC	48	48	48	48
E01 Hydroelectric VAUD	0	0	0	0
E02 Hydroelectric GENEVA	24	24	24	24
E00 Veytaux pumped storage	0	50	50	50
E08 Wind	0	0	0	0
Decentralised Generation Technologies	WN	WP1	WM	WP2
E07 PHV solar panels	0	0	0	0
RCC CHP combined heat power	50	50	50	50

Table 6: Demand devices generating electric loads GW

	Feeder 1	Feeder 2
Industry		
NHT Food, textile, leather, wood, paper, editing (conventional load)	0.182	-
CHT Chemical, rubber, glass, stone, metal (conventional load)	0.229	-
MAT Fabrication of machines, equipment, instruments (conventional load)	0.173	-
ALT Others (conventional load)	0.055	-
COT Construction (conventional load)	0.085	-
TRT Services (conventional load)	0.626	-
Electricity - Residential		
R11 El. appliances (conventional load)	0.268	0.302
RCL Lighting appliances (conventional load)	0.293	0.335
Heat - Existing Buildings 2-9 appts.		
RAT Heat pump old buildings (flexible load)	0.0231	-
Heat - Existing Houses		
RBT Heat pump old houses (flexible load)	-	0.0208
Heat - New Buildings 2-9 appts.		
RCT - Heat pump new buildings (flexible load)	-	0.0230
Heat New Houses (PJ/year)		
RDT Heat pump - new houses (flexible load)	-	0.0198
Hot Water - Buildings		
RE1 Residential w.h. solar/elec. (conventional load)	-	0.150
hot Water - Houses		
RF1 Residential w.h. solar/elec. (conventional load)	-	0.029
RFD hot water SFH, Solar (conventional load)	-	0.0002
Public Transport		
TB1 Tramway (conventional load)	0.0001	0.0001
TC1 Train (conventional load)	0.0112	-
Private Transport		
TES Electric car (flexible load)	0.1966	0.1193

Table 7: Loads at feeders GW

	Feeder 1				Feeder 2			
Conventional	WN	WP1	WM	WP2	WN	WP1	WM	WP2
NHT	0.0302	0.0507	0.0523	0.0493	-	-	-	-
CHT	0.0379	0.0636	0.0656	0.0618	-	-	-	-
MAT	0.0287	0.0480	0.0496	0.0467	-	-	-	-
ALT	0.0090	0.0152	0.0156	0.0147	-	-	-	-
COT	0.0141	0.0236	0.0243	0.0229	-	-	-	-
TRT	0.1152	0.1712	0.1721	0.1677	-	-	-	-
R11	0.0508	0.0789	0.1000	0.0380	0.1008	0.0789	0.0626	0.0600
RCL	0.0773	0.0475	0.1000	0.0679	0.1000	0.0475	0.0779	0.1100
RE1	-	-	-	-	0.0086	0.0488	0.0452	0.0474
RF1	-	-	-	-	0.0018	0.0095	0.0084	0.0095
RFD	-	-	-	-	0.000009	0.000075	0.000028	0.000084
TB1	0.000001	0.000035	0.000059	0.000041	0.000021	0.000040	0.000020	0.000041
TC1	-	0.0033	0.0034	0.0036	-	-	-	-
Flexible	WN	WP1	WM	WP2	WN	WP1	WM	WP2
RAT	0.0029	0.0026	0.0026	0.0029	-	-	-	-
RBT	-	-	-	-	0.0026	0.0026	0.0029	0.0030
RCT	-	-	-	-	0.0026	0.0026	0.0029	0.0030
RDT	-	-	-	-	0.0026	0.0026	0.0029	0.0030
TES	0.0006	0.0236	0.0283	0.0241	0.0050	0.0236	0.0070	0.0139

The energy demanded for electric car battery charging results from the choices made in the ETEM-SG scenario technology investment optimization.

Table 8 presents the installed capacity of solar panels (E07) and decentralized CHP units (RCC).

Table 8: Generation at feeders GW

	Feeder 1				Feeder 2			
Decentralized	WN	WP1	WM	WP2	WN	WP1	WM	WP2
E07	-	0.0020	0.0020	-	-	0.0020	0.0020	-
RCC	0.1100	0.0600	0.0400	0.0180	0.2200	0.1100	0.0700	0.0370

Optimal management of the distribution system. We present here the result of the optimization of the operation of this distribution system. Table 9 shows the power supplied, for each time slice, by each centralized generating unit connected at ∞-bus.

Table 9: Generation at ∞-bus GW

Real power	WN	WP1	WM	WP2
IMP	0.1813	0.1825	0.3384	0.3168
E0F	-	-	-	-
E01	0.0315	0.0630	0.0630	0.0630
E02	0.0510	0.1020	0.1020	0.1020
E00	-	0.2084	0.1959	0.1979
E08	-	0.0239	0.0242	0.0259
Reactive power	WN	WP1	WM	WP2
Total	0.1749	0.1550	0.1625	0.1480
Reserve	WN	WP1	WM	WP2
IMP	0.0015	-	-	-
E0F	-	-	-	-
E01	0.0315	-	-	-
E02	0.0510	-	-	-
E00	-	0.0546	0.0671	0.0651
E08	-	-	-	-

Table 10 shows the reactive power generated by conventional loads and compensated by flexible loads and decentralized units.

Table 10: Reactive power generated (+)/compensated (-) at feeders GW

	Feeder 1				Feeder 2			
Conventional	WN	WP1	WM	WP2	WN	WP1	WM	WP2
NHT	-0.0091	-0.0152	-0.0157	-0.0148	-	-	-	-
CHT	-0.0114	-0.0191	-0.0197	-0.0186	-	-	-	-
MAT	-0.0086	-0.0144	-0.0149	-0.0140	-	-	-	-
ALT	-0.0027	-0.0045	-0.0047	-0.0044	-	-	-	-
COT	-0.0042	-0.0071	-0.0073	-0.0069	-	-	-	-
TRT	-0.0346	-0.0514	-0.0516	-0.0503	-	-	-	-
R11	-0.0152	-0.0237	-0.0300	-0.0114	-0.0302	-0.0237	-0.0188	-0.0180
RCL	-0.0232	-0.0142	-0.0300	-0.0204	-0.0300	-0.0142	-0.0234	-0.0330
RE1	-	-	-	-	-0.0026	-0.0147	-0.0136	-0.0142
RF1	-	-	-	-	-0.0005	-0.0029	-0.0025	-0.0029
RFD	-	-	-	-	-0.0000	-0.0000	-0.0000	-0.0000
TB1	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
TC1	-	-0.0010	-0.0010	-0.0011	-	-	-	-
Flexible	WN	WP1	WM	WP2	WN	WP1	WM	WP2
RAT	0.0029	0.0032	0.0032	0.0029	-	-	-	-
RBT	-	-	-	-	0.0026	0.0026	0.0022	0.0022
RCT	-	-	-	-	0.0031	0.0032	0.0029	0.0028
RDT	-	-	-	-	0.0022	0.0024	0.0021	0.0020
TES	0.0006	0.0236	0.0717	0.0241	0.0050	0.0236	0.0070	0.0342
Decentralized	WN	WP1	WM	WP2	WN	WP1	WM	WP2
E07	0.0100	0.0080	0.0080	0.0100	0.0100	0.0080	0.0080	0.0100

Table 11 shows the contribution of secondary reserves provided by heat pumps (RAT, ..., RDT) and electric cars (TES).

Table 11: Reserve at feeders GW

	Feeder 1				Feeder 2			
Flexible	WN	WP1	WM	WP2	WN	WP1	WM	WP2
RAT	0.0029	0.0026	0.0026	0.0029	-	-	-	-
RBT	-	-	-	-	0.0026	0.0026	0.0022	0.0022
RCT	-	-	-	-	0.0026	0.0026	0.0029	0.0028
RDT	-	-	-	-	0.0022	0.0024	0.0021	0.0020
TES	0.0006	0.0236	0.0283	0.0241	0.0050	0.0236	0.0070	0.0139

Finally, in Table 12 we report the power losses in the distribution network.

Table 12: Real and reactive power losses in % of apparent power

	Feeder 1				Feeder 2			
	WN	WP1	WM	WP2	WN	WP1	WM	WP2
Real power	1.1	2.0	2.3	2.0	0.1	0.4	0.5	0.7
Reactive power	2.7	4.8	5.7	4.9	0.2	0.5	0.6	1.0

Remarks. The following features of the optimal solution are apparent:

1. The cost minimizing solution realizes the full potential of distributed generation and resources (EVs, heat pumps, storage and the like) to provide secondary reserves and demand response.
2. The flexible loads and electric cars can provide a significant proportion of the secondary reserves needed to compensate for intermittent and volatile renewable generation.
3. Power electronics such as inverters accompanying solar panels and chargers accompanying electric cars can compensate 20% (at night WN) to 64% (in the afternoon WM) of the reactive power consumed by inflexible loads.

This synergistic role of flexible loads is a major facilitator of efficiently integrating variable energy generation in the grid of the near future.

5. Conclusion and perspectives

In this paper we have adapted the non-linear load flow distribution market clearing approach of [36] to a computationally efficient linear programming approximation and have extended it to model flexible space conditioning loads and secondary reserves. We have shown that a straightforward linearization with one or two iterations to improve on the linearization gap can provide an accurate representation of market-based-marginal-cost-pricing incentives. Flexible loads and Distributed Energy Resources can respond to marginal cost based prices to provide demand-response, secondary reserves, and reactive power compensation. In numerical illustrations of the model, we have shown that these effects are non-negligible and we concluded that they should be taken into account in the regional integrated energy models that are currently developed in several countries.

In [10] the authors introduced constraints on regional power systems with high penetration of renewables, based on network reliability indices that were derived from an evaluation of the kinetic and magnetic potential storage in rotating generators, and the impact of an increasing share of non-rotating renewable generation. In the absence of additional fast reserves, they concluded that an empirical rule should be enforced, limiting to 30 % the maximum generation from renewables in non-interconnected zones. In our approach, developed in section 2, we address directly the fact that renewable generation exceeding a certain threshold results in a considerably higher demand for fast secondary reserves. Rather than limiting the integration of renewables beyond that threshold, we explicitly model additional reserve requirements and allow DERs and flexible loads to provide the requisite additional reserves, and thus deal with the reliability concern addressed in [10]. We conclude by noting that the interesting feature of smart grid distribution systems is that they allow the provision of stability sustaining reserve to be provided by the load side. This synergistic complementarity of renewable generation and flexible loads must be modeled and accounted for in the regional integrated energy models, like TIMES [6], OSeMOSYS [17, 46], or ETEM-SG [3]. The linear programming formulation presented here can be easily included in these models.

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